> A **combination** counts the number of ways *n different* objects can be formed into groups of size *s* (in these cases, order of arrangement or selection is *not* important).*
>
> $$C_{n,s} = \frac{n!}{(n-s)!\,s!}$$

Example — From 5 books (A, B, C, D and E), how many reading lists of 3 different books can be made up?

Solution — Well, if order of arrangement does *not* count, meaning reading list A, B, C is considered no different from reading list A, C, B (same books), we use the combination formula, as follows.

$$C_{n,s} = \frac{n!}{(n-s)!\,s!}$$

$$C_{5,3} = \frac{5!}{(5-3)!\,3!} = \frac{5!}{2!\,3!}$$

$$C_{5,3} = \frac{(5)(4)(3)(2)(1)}{(2)(1)\,(3)(2)(1)}$$

$$C_{5,3} = 10 \text{ ways}$$

10 Ways Listed

ABC	BCD	CDE
ABD	BCE	
ABE	BDE	
ACD		
ACE		
ADE		

In other words, only 10 reading lists are possible. Keep in mind, although 60 arrangements of three books are possible (if order counts), only 10 of these arrangements involve groups containing different books. ■

Summary

The study of probability grew out of the gambling dens of Europe over three centuries ago. Much of what we study today is still based on this early work.

In broad terms, we define probability as follows.

Probability: The proportion (or percentage) of times an event will occur in the long run, under similar circumstances.

Two methods we can use to obtain this probability percentage are the empirical approximation and classical approach.

Empirical approximation to probability: The fraction of times an event *has actually occurred* over a great many experiments conducted over a long period of time under similar circumstances, expressed as,

$$P(\text{an event}) \approx \frac{\text{Number of Times Event Has Actually Occurred}}{\text{Total Number of Experiments (or Attempts)}}$$

As the number of experiments increases, this empirical fraction gets closer and closer to the true probability.

*Combinations may also be denoted as ${}_nC_s$, $C(n,s)$, C^n_s, and $\begin{bmatrix} n \\ s \end{bmatrix}$.

Dependent events: The occurrence or nonoccurrence of one event affects the probability of one or more other events.

Independent events: The occurrence or nonoccurrence of one event in no way affects the probability of any other event.

Because many of the formulas we use later in the text when we sample require the condition of independence, it is important when we design a study to do our best to ensure independence of events when we can.

Counting principle: For a sequence of events, in which the first event can occur in a ways, the second event in b ways, the third event in c ways, and so on, the total number of ways the events can occur together is

$$a \cdot b \cdot c \ldots$$

Early gambling experiments leading to discovery of the normal curve: Early gambling experiments (usually involving the tossing of coins and dice) form the theoretical underpinning of many of the formulas and statistical procedures we use today in statistics (such as chi-square analysis and tests of proportions, which are discussed at length in chapters 10 and 11). However, these early experiments also paved the way for one of the most important discoveries in statistics and the subject of the next chapter, the normal curve.

Exercises

4-22-08 3.1 - 3.4 (sk.p 3.3e)

Note that full answers for exercises 1–5 and abbreviated answers for odd-numbered exercises thereafter are provided in the Answer Key.

3.1 Out of 200 tosses of a dart at a Friday night tournament, a player strikes the bull's-eye 50 times.

a. What is the true probability of a bull's-eye for this player?
b. Explain how one might empirically determine the true probability of a bull's-eye for this player.

3.2 Suppose you attend a party of 50 people (30 men and 20 women). If 10 of the men and 5 of the women have curly black hair and you were to *randomly* select one person from the party, what is the probability this one person would

a. have curly black hair?
b. be male?
c. be male and have curly black hair?
d. be male *or* have curly black hair?
e. be female and *not* have curly black hair?

Given we already know the person selected is a male, what is the probability the person selected

f. has curly black hair?
g. is a female?

3.3 Suppose we attend a party of 5 people, of which 2 are famous TV celebrities. What if we were to randomly select 2 people from this party.

a. Use a tree diagram to list all equally likely ways two people may be selected. Assume the first person is removed from the party prior to the second pick. Hint: use numbers 1, 2, and 3 to identify the *non*-TV celebrities and the numbers 4 and 5 to identify the TV celebrities. Then find the probability
  i. Both are TV celebrities.
  ii. Exactly one is a TV celebrity.
  iii. None are TV celebrities.

b. Assume the first person *is* replaced prior to the second pick. List the new sample space of equally likely outcomes, then find the probability
  i. Both are TV celebrities.
  ii. Exactly one is a TV celebrity.
  iii. None are TV celebrities.

c. Use the multiplication rule to calculate the probability *both* are TV celebrities if the first person is
  i. removed after the first pick.
  ii. replaced after the first pick.

d. Use the multiplication rule to calculate the probability *none* are TV celebrities if the first person is
  i. removed after the first pick.
  ii. replaced after the first pick.

3.12 A store rents out VCRs. Out of 7 available for rental, 3 have defects. If 2 VCRs are rented together, find the probability

a. both will be defective.
b. neither will be defective.

Challenging Question:

c. only one will be defective.

3.13 Out of three cards selected from a deck of 52, what is the probability all three will be aces? Solve without replacement, then solve with replacement.

3.14 Suppose we roll a pair of dice. List the sample space of equally likely outcomes, then find the probability of getting

a. snake eyes (two 1's).
b. a 5 on one die and a number other than 5 on the other die.
c. a sum of 7.

3.15 If we toss a coin 5 times, use the special multiplication rule to find the probability of getting 5 heads.

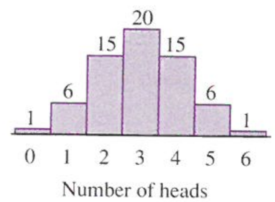
3.16 There are 3 roads from Allentown to Boynton. How many different round trips are possible if a person

a. *must* take a different road on the return trip?
b. can take either road either way?
c. *must* take the same road both ways?

3.17 A company employs 7 salespeople, 10 office workers, and 3 administrators. How many different committees can be formed consisting of a salesperson, an office worker, and an administrator?

3.18 Six coins are shaken in a tin can then dropped on a table and the number of heads counted. If we repeat the experiment a large number of times, how many times would we expect to get 0 heads, 1 head, 2 heads, 3 heads, 4 heads, 5 heads, and 6 heads? We can actually perform this experiment over and over again thousands of times to get the answer, or an easier way would be to use a tree diagram to help us list the sample space of equally likely outcomes.

Let's say the sample space was listed for us. This would result in 64 equally likely outcomes, which when tallied according to the number of heads would appear as follows:

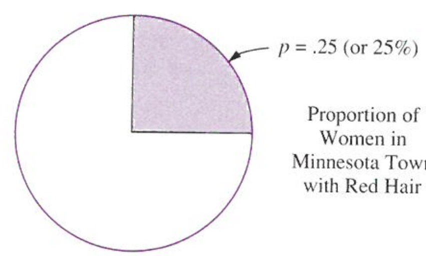


Expected Number of Heads Achieved if 6 Coins Are Tossed 64 Times

Based on the histogram, if you were to toss a coin six times, what is the probability of getting

a. 0 heads (in other words, all tails)?
b. 3 heads?
c. 4 or more heads?

3.19 It is known 25% of the women in a certain town in Minnesota possess the attribute of red hair and this might be expressed as follows:



Proportion of Women in Minnesota Town with Red Hair

If we were to randomly sample 60 women from this town, how many would you expect to have red hair?

3.20 An experiment has outcomes 0, 1, 2, 3, and 4 with the probabilities as shown:

$p(x)$:	.3	.3	.2	.1	?
x:	0	1	2	3	4

For this probability distribution,

a. Fill in the missing $p(x)$ value.
b. Construct the probability histogram.
c. Calculate μ and σ.